

These notes represent incomplete work in progress towards a space cable design that can launch small unmanned vehicles directly to interplanetary space.

Introduction

In the Space Cable, momentum is transferred to a vehicle from the bolts travelling in the tubes. The bolts carry powerful permanent magnets, and these interact with the coils in the vehicle's bearer to form a kind of linear electric generator.

There are two modes of operation.

1. The vehicle is travelling below the speed of the bolts. The bearer extracts a fraction of the power (up to 10%) and momentum (up to 5%) from the rising bolts as they pass. There is some spare power that can be used to accelerate the descending bolts and gain some extra momentum. This is a useful way to dispose of the excess power.
2. The vehicle is travelling faster than the bolts. The bearer moderately retards the descending bolts to generate power and uses this to retard the rising bolts. Although retarding the descending bolts creates unwanted drag on the bearer and vehicle, the thrust gained by retarding the rising bolts is greater, and there is enough thrust to achieve high acceleration of masses of a few tons.

When the speeds are close, the operation is ineffective, since the bolts are passing the vehicle at a low relative speed; there are not enough bolts passing in a given time to transfer a useful amount of energy or momentum. For human travel, it is necessary to keep the vehicle speeds below those of the bolts. As the calculations below show, it is possible to accelerate a 100 tonne vehicle at 40 m/sec^2 (i.e., $4g$) to 2 km/sec . This is similar or a little better than the performance of a first stage rocket. To get such a vehicle into orbit, it must carry rockets.

For small unmanned vehicles, accelerations of $400g$ are feasible. For this application, a two-speed arrangement of bolts is necessary in which some tubes carry bolts at twice the speed of the others. The load-support requirements can be balanced by ensuring that the slower bolts are heavier than the faster ones. As the vehicle and bearer catch up with the slower bolts, they can only gain useful thrust from the faster ones. As they catch up with the faster bolts, they overtake the slower ones and they can switch to using them. Eventually the speed exceeds that of both sets of bolts and bearer can use them all again.

The limits come in the efficiency of the electric generator and motor within the bearer. A bearer as long as 500 metres is envisaged so as to dispose of excess heat, and its mass has to be subtracted from the useful payload that can be carried. This and other considerations remain to be investigated.

Calculations

The force on the vehicle and on the passing bolts must be equal and opposite, by Newton's law. The force is equal to the rate of change of momentum, and so these must also be equal. If M_v is the combined mass of the vehicle and its bearer and v is their velocity, the momentum conservation equation is as follows.

$$M_v \frac{dv}{dt} + m_b l \frac{dV}{dt} = 0$$

Here, m_b is the average mass per unit length of the bolts, and V is the bolts' velocity. The length l of the bearer determines how many bolts interact with it and the launch vehicle.

There is also an exchange of energy, but this leads to an excess of power. The power drawn from the passing bolts is

$$\frac{d}{dt} \left(\frac{1}{2} m_b l V^2 \right) = m_b l V \frac{dV}{dt}$$

The power needed to accelerate the vehicle is

$$\frac{d}{dt} \left(\frac{1}{2} M_v v^2 \right) = M_v v \frac{dv}{dt}$$

The ratio of these two powers is proportional to v/V . The vehicle obviously starts off very slowly and so $v < V$. The power drawn from the bolts is then much greater than that needed for accelerating the vehicle, and some means is needed to dispose of this excess power, which is likely to be considerable.

Otherwise, overheating of the electromagnetic circuits will become a serious problem. The most productive use of the excess power is to accelerate the descending bolts as well as retarding the rising ones. At first, the vehicle is climbing slowly, and the additional thrust gained by accelerating the descending bolts is substantial. The advantage tails off as the vehicle gains speed, but it remains a useful way of disposing of excess power.

Let the respective velocities of the rising and descending bolts be V_r and $-V_d$, taking both variables as positive. The energy conservation equation is then

$$M_v v \frac{dv}{dt} + m_b l \left(V_r \varepsilon_r \frac{dV_r}{dt} + V_d \varepsilon_d \frac{dV_d}{dt} \right) = 0$$

The terms ε_r and ε_d represent the efficiency of the linear electric generator and motor and will be less than one. The momentum conservation equation is

$$M_v \frac{dv}{dt} + m_b l \left(\frac{dV_r}{dt} - \frac{dV_d}{dt} \right) = 0$$

Eliminate the derivative of V_d to obtain

$$M_v (v + V_d \varepsilon_d) \frac{dv}{dt} + m_b l (V_r \varepsilon_r + V_d \varepsilon_d) \frac{dV_r}{dt} = 0$$

Similarly

$$M_v (v - V_r \varepsilon_r) \frac{dv}{dt} + m_b l (V_d \varepsilon_d + V_r \varepsilon_r) \frac{dV_d}{dt} = 0$$

That is

$$\begin{aligned} \frac{dv}{dt} &= - \frac{m_b l (V_r \varepsilon_r + V_d \varepsilon_d)}{M_v (v + V_d \varepsilon_d)} \frac{dV_r}{dt} \\ &= - \frac{m_b l (V_r \varepsilon_r + V_d \varepsilon_d)}{M_v (v - V_r \varepsilon_r)} \frac{dV_d}{dt} \end{aligned}$$

Example 1:

Bolt mass $m_b = 2$ kg/metre, bearer length $l = 500$ metres, $V_r = V_d = 4.5$ km/sec, $v = 2$ km/sec, and efficiency $\varepsilon_r = \varepsilon_d = 0.6$. Acceleration needed over $d = 50$ km: $\frac{dv}{dt} = \frac{v^2}{2d} = \frac{4 \times 10^6}{2 \times 5 \times 10^4} = 40$ m/sec². Allow the rising bolts to lose about 5% of their energy while in contact with the bearer, which they traverse in time $\frac{l}{V_r - v}$, where $V_r - v$ is the relative velocity of the bolts with respect to the bearer. This equates to

approximately 2.5% of their velocity, a reduction of about 0.13 km/sec. This gives acceleration

$$\frac{dV_r}{dt} = -0.13 \times 10^3 \frac{V_r - v}{l} = -0.13 \times 10^3 \frac{2.5 \times 10^3}{5 \times 10^2} \approx -6 \times 10^2 \text{ m/sec}^2.$$

$$\text{Use } \frac{dv}{dt} = - \frac{m_b l (V_r \varepsilon_r + V_d \varepsilon_d)}{M_v (v + V_d \varepsilon_d)} \frac{dV_r}{dt}. \text{ Then } \frac{dv}{dt} = \frac{2 \times 0.6 (4.5 + 4.5) 10^3 l \cdot 0.13 \times 10^3 \times 2.5 \times 10^3}{(2 + 0.6 \times 4.5) 10^3 M_v} \approx \frac{7.5 \times 10^5}{M_v}.$$

Therefore $M_v = \frac{7.5 \times 10^5}{40} \approx 1.9 \times 10^4$ kg or 19 tonnes. This is per tube pair, and so five tube pairs can launch 95 tonnes.

Example 2:

$m_b = 2$ kg/metre, $l = 500$ metres, $V_r = -V_d = 4.5$ km/sec, $v = 20$ km/sec, and efficiency $\varepsilon_r = \varepsilon_d = 0.6$.

Acceleration needed over $d = 50$ km: $\frac{dv}{dt} = \frac{v^2}{2d} = \frac{4 \times 10^8}{2 \times 5 \times 10^4} = 4000$ m/sec². Allow the rising bolts to lose

0.13 km/sec of velocity while in contact with the bearer, which corresponds to a traversal time of $\frac{l}{v - V_r}$, where $v - V_r$ is the relative velocity of the bolts with respect to the bearer. This gives

acceleration $\frac{dV_r}{dt} = -0.13 \times 10^3 \frac{v - V_r}{l} = -0.13 \times 10^3 \frac{15.5 \times 10^3}{5 \times 10^2} \approx -4 \times 10^3$ m/sec². Take the acceleration of

example 1 as using maximum thrust and use $\frac{dv}{dt} = - \frac{m_b l (V_r \varepsilon_r + V_d \varepsilon_d)}{M_v (v + V_d \varepsilon_d)} \frac{dV_r}{dt}$.

Then $\frac{dv}{dt} = \frac{2 \times 0.6(4.5 + 4.5)10^3 l}{(20 + 0.6 \times 4.5)10^3 M_v} \frac{0.13 \times 10^3 \times 15.5 \times 10^3}{l} \approx \frac{9.6 \times 10^5}{M_v}$. Therefore $M_v = \frac{9.6 \times 10^5}{40} \approx 2.4 \times 10^4$ kg or 24 tonnes.

Conclusion

Earlier papers have calculated the maximum thrust obtainable from a single bolt as being in the kilo Newton range, but more detailed work on that is needed. Another important area to be investigated is whether the stability mechanisms designed to cope with cross winds are adequate to deal with the disturbances caused by launching vehicles. The extra complexity of having two bolt speeds should be factored into the cost.

These notes show the feasibility of the basic principle of accelerating vehicles to orbital and interplanetary speeds without needing the bolts to travel at those speeds.