

IMPROVING STABILITY OF THE SPACE CABLE

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ABSTRACT

The space cable is a variation on the *launch loop*. It consists of evacuated tubes supported by fast-moving objects, called *bolts*, travelling inside them. The bolts use magnetic levitation to support the tubes. Version 1 was designed to replace the role of a first-stage rocket by lifting space vehicles from the ground to 50 km at 5800 km/hour. Version 2 rises to 140 km and has wider applications.

A key issue with such dynamically supported structures is lateral stability, particularly in the presence of varying cross winds in the stratosphere. A solution has now been found to the relevant partial differential equation that makes it possible to ensure stability without the need for heavy tethers and pipes. Rather, the electromagnets in the bolts can be used with suitable electronic control. The ancillary support structures only need to be about 400 metres high, a factor of 35 reduction over the previous solution.

INTRODUCTION

The space cable¹ is a variant of the launch loop,² which is itself a variant of the space elevator.³ The space cable is supported by fast-travelling projectiles, called *bolts*. The bolts travel inside tubes, which are evacuated. To eliminate friction, the bolts are coupled to the tubes using magnetic levitation.

The space cable is supported from two surface stations, either on the land or at sea. Two versions have been described. Version 1 rises to a height of 50 km and covers a range between the two surface stations of 150 km. Version 2 reaches a height of 140 km over approximately the same range. Version 2 is about three times as expensive to build as Version 1. Both are considerably smaller than the launch loop, which requires a range of 2000 km.

The main purpose of the space cable is to reduce the cost of launching vehicles and payloads into orbit. Version 1 is designed to replace the first stage of a rocket or to support vehicles capable of getting to orbit in a single stage. During launch, a small fraction of the kinetic energy of the bolts is transferred to the vehicle. This process can accelerate it to a velocity of 5800 km/sec at a height of 50 km. Version 2 can achieve greater height and velocity.

Like the space elevator, the space cable can be used to support fixed installations such as communications facilities or an astronomical observatory. In addition, there is likely to be a tourist market for rides on the cable at much gentler speeds than are needed for launching using rockets.

Both the space cable and the launch loop have the advantage that they can be built with materials that are already available. A significant remaining challenge is to control lateral movement. Previously, a scheme was

described that used some fairly heavy supports and tethers,⁴ but now a new technique called *active curvature control* can be presented.

SUMMARY OF OPERATION

Each bolt travels along one tube from one surface station to the other. There, the bolt is turned around and sent back along another tube. On arrival at the surface station, it is again turned around. This continues indefinitely through a pair of adjacent tubes. Typically there will be a bolt every five metres. As shown in Figure 1, the magnetic levitation force from the bolts is perpendicular to their direction of travel; this combines with tension in the tubes to support the structure. To make the space cable robust, several pairs of tubes are used, and there are arrangements for quiescing one pair for maintenance while the others continue operating.

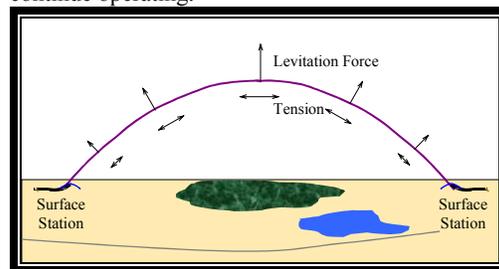


Figure 1 Space Cable Layout

Previous publications^{1,5} have described the preferred arrangement of permanent magnets in the tubes and the bolts, which together provide a form of magnetic levitation. There are electronically controlled electromagnets to stabilize this levitation. Since the electromagnets are only for stabilization, the energy consumption is low. Other experience of similar mixes of permanent magnets and electromagnets in magnetic bearings has confirmed this.⁶

Overall Balance of Forces

At each surface station, there has to be enough movement of the cable supports to accommodate tilting that is sufficient to counterbalance lateral forces on the cable. The principal source of lateral forces is from cross winds, and the cable copes with these through controlled bending.

It is both desirable and possible to keep the upper part of the space cable in a stable position. At high altitudes, there is little or no wind. Maintaining a stable position here increases the usefulness of astronomical and other instruments and simplifies the trajectory of space vehicles during launch.

ACTIVE CURVATURE CONTROL

Any bend or bulge in a tube will cause turning of the bolts travelling inside. This turning creates a centrifugal force. Uncontrolled, the centrifugal force will increase the bend or bulge indefinitely, leading to catastrophic instability. However, careful control allows the centrifugal force to be exploited so as to counteract external forces, mainly wind.

The air is calm at high altitudes (above 12-15 km) and there are no other lateral forces, but there are cross winds lower down. In calm air, the space cable has no lateral bending, but it does bend in the windy region. The technique is to measure the wind speed using a form of anemometer and adjust the bending so as to counteract the wind exactly. At the boundary between these two regions, we see (as in Figure 2) that the bending causes a change in the gradient of the space cable lower down (i.e., nearer the surface station) but not higher up. Part of active curvature control is ensuring that the gradient is altered only in the lower part of the space cable and not in the higher part. In this way, the lateral force is transmitted down to the surface station and not upwards. At the surface station, this force can be absorbed by means of an opposite bend, and the higher part remains stable.

To adjust the curvature over a short length of tube, we use electronically controlled actuators. The preferred design is to use the bolts' own electromagnets (see Section "Design of Actuators"). In the case of a change of wind speed, the wind itself causes some bending, and the purpose of the actuators is to limit and direct that bending. As an example, consider the sudden onset of a wind below (i.e., to the left of) point P in Figure 2 with calm air above P . The wind bends the tubes at P , and that bend opposes the wind. However, if it were left unchecked, the tubes would continue to bend at or near P in excess of the curvature that balances the wind, creating instability. The actuators slow the bending until the curvature matches the wind.

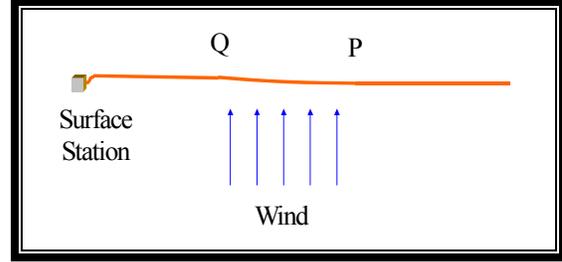


Figure 2 View of Lateral Displacement

Below P , this action has the effect of causing further bending; at each point, the actuators limit the bending to a curvature that balances the wind. Thus the actuators maintain stability; they minimize the expenditure of energy by exploiting the effect of the wind in a controlled way. They exert forces that restrain bending and spread it downwards from the initial stress point at P .

Suppose that below point Q there is no wind. Despite the lack of wind, it is still necessary to propagate the deflection in the lateral gradient of the tubes down to the surface station and avoid such propagation upwards from P . This is achieved by local point-to-point telemetry downwards along the tubes. Each wind sensor communicates estimates of the required lateral gradient and the actual lateral gradient to the neighbouring actuator below it. It calculates this by receiving the required and actual lateral gradients from the neighbouring actuator above it and adding the effect of its own required curvature and actual curvature. Similarly, each sensor communicates estimates of the required and actual lateral displacement to the neighbouring actuator below it. The actuator then adjusts the curvature in order to move the tubes towards the required displacement. As the gap closes between required and actual displacement, the estimates can be refined, since the curvatures and gradients can be calculated more accurately. As the displacement approaches the required value, the actuators reduce their curvature until the tubes are again straight but at the required angle all the way down to the surface station.

The next section gives the mathematical derivation for the operation of the actuators. Section "Some Typical Numbers" applies the formulas to the example illustrated in Figure 2. After that, there are some design details, followed by conclusions.

MATHEMATICAL DERIVATION

The lateral displacements z_1 and z_2 of a pair of tubes in which bolts travel in opposing directions have been shown⁴ to obey the following partial differential equations:

$$m_u \frac{\partial^2 z_1}{\partial t^2} + 2m_b V \frac{\partial^2 z_1}{\partial x \partial t} + (m_b V^2 - Tu) \frac{\partial^2 z_1}{\partial x^2} = F + F_s \quad (1)$$

$$m_u \frac{\partial^2 z_2}{\partial t^2} - 2m_b V \frac{\partial^2 z_2}{\partial x \partial t} + (m_b V^2 - Tu) \frac{\partial^2 z_2}{\partial x^2} = F - F_s \quad (2)$$

Here, t is time and x is distance along the tubes. The equations assume that the tubes are connected by struts spaced u apart that exert an equal and opposite force F_s on each tube. Each section of tube of length u is subject to an external force F . The tubes carry bolts of average mass m_b per distance u travelling in opposing directions at velocity V . The combined mass of a bolt and tube over distance u is m_u . There is a tension T in each tube.

We now dispense with the struts and join the two tubes together. Then $z_1 = z_2 = z$ and we obtain a single equation by summing, slightly simplified by setting $u=1$ so that F , m_b and m_u are taken over unit distance:

$$m_u \frac{\partial^2 z}{\partial t^2} + (m_b V^2 - T) \frac{\partial^2 z}{\partial x^2} = F \quad (3)$$

This equation makes clear that the curvature term $(m_b V^2 - T) \frac{\partial^2 z}{\partial x^2}$ can balance the force F and eliminate

any acceleration $\frac{\partial^2 z}{\partial t^2}$. Write $c = \frac{\partial^2 z}{\partial x^2}$ and

$M = m_b V^2 - T$. Remove the subscript u in m_u . Twice partially differentiate equation (3) with respect to x to obtain the curvature equation:

$$m \frac{\partial^2 c}{\partial t^2} + M \frac{\partial^2 c}{\partial x^2} = B + B_a \quad (4)$$

M has been taken as constant, since the variation due to gravity is small compared to the wind forces now under consideration.

$B = \frac{\partial^2 F}{\partial x^2}$ is the *bending stress* caused by variation in

the external force along the tubes, and B_a is an extra effect, the *bending compensation* due to the actuators.

For any pattern of wind, there is a stable arrangement of the cable. To see this, consider a required displacement z_r that satisfies for all x the equation

$$M \frac{\partial^2 z_r}{\partial x^2} = F \quad (5)$$

From equation (3) we see that $\frac{\partial^2 z_r}{\partial t^2} = 0$, and so the position is stable. Introduce the required curvature

$$c_r = \frac{\partial^2 z_r}{\partial x^2} \text{ so that equation (5) can be written}$$

$$M c_r = F \quad (6)$$

To restore stability when the wind changes, it is necessary to move the cable to the new stable arrangement. As Figure 3 illustrates, the wind itself tends to push the cable towards the stable position. The job of the control mechanism is to limit the movement and spread it more evenly, using the curvature control to ensure that the cable's curvature reaches but does not overshoot a value that satisfies equation (6) at every

point. To achieve this goal, the actuators adjust the bending compensation B_a to cause the cable's curvature c to change. It is convenient to define a compound bending quantity B_c such that

$$B_c = B_a - \frac{m}{M} \frac{\partial^2 F}{\partial t^2} \quad (7)$$

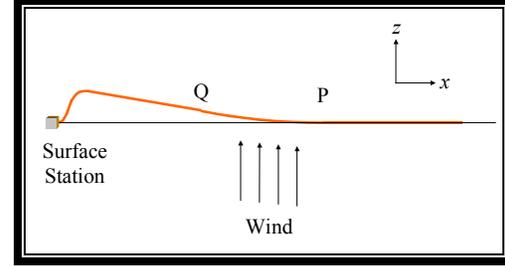


Figure 3 Exaggerated View of Lateral Displacement

The bending quantity B_c incorporates the external effect caused by the wind varying along the tube's length. We assume that B can be estimated from the wind force F . Similarly we assume that the acceleration in wind force $\frac{\partial^2 F}{\partial t^2}$ can be estimated.

From equations (4) and (7)

$$B_c = m \frac{\partial^2 c}{\partial t^2} - \frac{m_u}{M} \frac{\partial^2 F}{\partial t^2} + M \frac{\partial^2 c}{\partial x^2} - B \quad (8)$$

By the definition of B , this is

$$B_c = m \frac{\partial^2 c}{\partial t^2} - \frac{m_u}{M} \frac{\partial^2 F}{\partial t^2} + M \frac{\partial^2 c}{\partial x^2} - \frac{\partial^2 F}{\partial x^2}$$

Hence and from equation (6)

$$B_c = m \frac{\partial^2}{\partial t^2} (c - c_r) + M \frac{\partial^2}{\partial x^2} (c - c_r) \quad (9)$$

Now we introduce an intermediate curvature c_c . The intermediate curvature is designed to cause the cable to move itself from displacement z to the required displacement z_r . It incorporates the wind force and a constant λ , which is subject to certain constraints that are to be determined. It is defined as follows.

$$c_c = c_r - \lambda(z - z_r) \quad (10)$$

The definition of c_c allows for the effect of wind but also addresses the case where a lower part of the cable must move from displacement z to z_r to accommodate wind higher up. As the cable moves from z to z_r , the intermediate curvature converges to the required curvature c_r .

It is therefore necessary to set B_a so that the curvature is drawn to the intermediate value c_c . This is achieved by setting the following value for B_c from which B_a can be derived by equation (7).

$$B_c = -\varepsilon(c - c_c) - \kappa \frac{\partial}{\partial t} (c - c_r) \quad (11)$$

The final term with a factor κ is for damping. For later simplicity, it is based directly on the target curvature c_r rather than c_c . The constants ε and κ are

subject to certain constraints to be determined. The minus signs have been chosen for later convenience.

Where the wind is already blowing the cable towards displacement z_r , the curvature has to adjust to oppose the wind, as desired. At lower parts of the cable, if there is less wind, it is necessary to bend it deliberately so that it aligns to support the region under wind stress.

Simplify equation (10) by substituting $z' = z - z_r$ to give

$$c_c = c_r - \lambda z' \quad (12)$$

Substituting into equation (11) yields

$$B_c = -\varepsilon(c - c_r) - \varepsilon \lambda z' - \kappa \frac{\partial}{\partial t}(c - c_r) \quad (13)$$

Since $z' = z - z_r$, it follows that

$$\frac{\partial^2 z'}{\partial x^2} = \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z_r}{\partial x^2} = c - c_r \quad (14)$$

Therefore

$$B_c = -\varepsilon \frac{\partial^2 z'}{\partial x^2} - \varepsilon \lambda z' - \kappa \frac{\partial^3 z'}{\partial t \partial x^2} \quad (15)$$

We also have, from equations (9) and (14)

$$B_c = m \frac{\partial^4 z'}{\partial t^2 \partial x^2} + M \frac{\partial^4 z'}{\partial x^4} \quad (16)$$

Equations (15) and (16) combine to give the final equation governing the operation of the active curvature control mechanism as follows.

$$m \frac{\partial^4 z'}{\partial t^2 \partial x^2} + M \frac{\partial^4 z'}{\partial x^4} + \varepsilon \frac{\partial^2 z'}{\partial x^2} + \varepsilon \lambda z' + \kappa \frac{\partial^3 z'}{\partial t \partial x^2} = 0 \quad (17)$$

Solution to the Curvature Control Equation

Equation (17) has solutions of the form $z' = \gamma e^{i\mu t + \phi x}$ leading to the following equation in ψ and ϕ

$$m \phi^2 \psi^2 + M \phi^4 + \varepsilon \phi^2 + \varepsilon \lambda + \kappa \phi^2 \psi = 0 \quad (18)$$

Equation (18) expresses the relationship between ψ and ϕ , and so it represents a family of solutions. The general solution may be written as a continuous integral, treating ψ as a function of ϕ .

$$z'(x, t) = A \int_{c-i\infty}^{c+i\infty} Z_L(\phi) e^{i\mu t} e^{\phi x} d\phi \quad (19)$$

This has the form of an inverse Laplace transform,⁷ which is closely related to the Fourier transform but allows for ϕ to be complex. The limits have been taken on the assumption that ϕ is complex. The integral has to follow a contour in the complex plane, and this has the form of a real constant c plus an imaginary value ranging between $\pm\infty$. The constant A in the Laplace transform is $A = 1/2\pi i$, where $i = \sqrt{-1}$.

The quantities ψ and ϕ are complex in general. To obtain solutions that converge over time, the real part

of ψ in equation (19) should be negative. Equation (18) is quadratic in ψ with the solution

$$\psi = \frac{-\kappa \phi^2 \pm \sqrt{\kappa^2 \phi^4 - 4m \phi^2 (M \phi^4 + \varepsilon \lambda + \varepsilon \phi^2)}}{2m \phi^2}$$

This simplifies to the form

$$2m \psi = -\kappa \pm \sqrt{\kappa^2 - 4m(M \phi^2 + \varepsilon \lambda \phi^{-2} + \varepsilon)} \quad (20)$$

The real part of ψ is definitely negative if κ is real, $\kappa > 0$, and

$$\left| \sqrt{\kappa^2 - 4m(M \phi^2 + \varepsilon \lambda \phi^{-2} + \varepsilon)} \right| < \kappa$$

It is helpful to insert a factor H as follows.

$$\left| \sqrt{\kappa^2 - 4m(M \phi^2 + \varepsilon \lambda \phi^{-2} + \varepsilon)} \right| < H \kappa \quad (21)$$

H is between 0 and 1 and permits control over the speed of convergence by ensuring that ψ is not too near to zero while remaining negative. In fact,

$$2m \psi < -\kappa(1-H) \quad (22)$$

For example if $H = 0.9$, $\psi < -\kappa/20m$.

Details of Convergence Condition

Inequality (21) is true if

$$\left| \kappa^2 - 4m(M \phi^2 + \varepsilon \lambda \phi^{-2} + \varepsilon) \right|^2 < H^4 \kappa^4 \quad (23)$$

Let $\phi = \Phi e^{i\mu}$ with Φ and μ real. Then

$$\phi^2 = \Phi^2 e^{2i\mu} = \Phi^2 (\cos 2\mu + i \sin 2\mu)$$

$$\text{and } \phi^{-2} = \Phi^{-2} e^{-2i\mu} = \Phi^{-2} (\cos 2\mu - i \sin 2\mu)$$

The constants κ, ε and λ are real, so inequality (23) becomes

$$\begin{aligned} & (\kappa^2 - 4m(M \Phi^2 + \varepsilon \lambda \Phi^{-2}) \cos 2\mu - 4m\varepsilon)^2 \\ & + (4m(M \Phi^2 - \varepsilon \lambda \Phi^{-2}) \sin 2\mu)^2 < H^4 \kappa^4 \end{aligned}$$

This expands as follows.

$$\begin{aligned} & 16m^2 (M \Phi^2 + \varepsilon \lambda \Phi^{-2})^2 (\cos^2 2\mu + \sin^2 2\mu) \\ & - 64m^2 M \varepsilon \lambda \sin^2 2\mu \\ & - 8m(\kappa^2 - 4m\varepsilon)(M \Phi^2 + \varepsilon \lambda \Phi^{-2}) \cos 2\mu \\ & - 8m\varepsilon \kappa^2 + 16m^2 \varepsilon^2 + \kappa^4 (1 - H^4) < 0 \end{aligned}$$

Write $G = M \Phi^2 + \varepsilon \lambda \Phi^{-2}$ and divide by $8m$.

$$\begin{aligned} & 2mG^2 + (4m\varepsilon - \kappa^2)G \cos 2\mu \\ & + \varepsilon(2m\varepsilon - \kappa^2 - 8mM\varepsilon\lambda \sin^2 2\mu) \\ & + \kappa^4 (1 - H^4)/8m < 0 \end{aligned} \quad (24)$$

Range of G

The quadratic expression in inequality (24) is positive for large G . It is therefore necessary to choose values for the constants κ, ε and λ that ensure that the expression is negative over the physically possible range of values of G . The minimum value of G is found by differentiating it with respect to Φ and taking

$$\frac{dG}{dF} = 2M\Phi - 2\varepsilon\lambda\Phi^{-3} = 0$$

When $\Phi^4 = \varepsilon\lambda/M$ this gives the minimum value for G of

$$G_{\min} = 2\sqrt{M\varepsilon\lambda} \quad (25)$$

The maximum of G depends on the maximum and minimum values of Φ . In the solution to equation (17), $z' = \gamma e^{v\phi + \phi z}$. Now write $\phi = E + 2\pi i/W$ to see that the complex value ϕ represents a standing wave in the cable of wavelength W modified by a real exponential term E giving $z' = \gamma e^{v\phi} e^{Ez} e^{2\pi i z/W}$ and

$$\Phi^2 = |\varepsilon|^2 = E^2 + \left(\frac{2\pi}{W}\right)^2$$

Therefore

$$G = M \left(E^2 + \frac{4\pi^2}{W^2} \right) + \frac{\varepsilon\lambda}{E^2 + 4\pi^2/W^2}$$

The maximum value of G depends on both the minimum and maximum values of W and E . There are two potential maxima G_2 and G_{-2} , and

$$G_{\max} = \text{Max}(G_2, G_{-2}) \quad (26)$$

$$G_2 = M \left(E_{\max}^2 + \frac{4\pi^2}{W_{\min}^2} \right) + \frac{\varepsilon\lambda}{E_{\max}^2 + 4\pi^2/W_{\min}^2} \quad (27)$$

$$G_{-2} = M \left(E_{\min}^2 + \frac{4\pi^2}{W_{\max}^2} \right) + \frac{\varepsilon\lambda}{E_{\min}^2 + 4\pi^2/W_{\max}^2} \quad (28)$$

Since both ends of the space cable are anchored, the maximum wavelength is twice the cable length. The minimum wavelength W_{\min} is of the same order as the size of an active element of an actuator. The largest exponent E_{\max} is found by considering the greatest deflection D_{\max} that can occur over the range x_R at which winds operate:

$$D_{\max} = e^{x_R E_{\max}}$$

Thus

$$E_{\max} = \frac{1}{x_R} \ln D_{\max} \quad (29)$$

The smallest exponent E_{\min} is zero, which enables equation (28) to be simplified as follows.

$$G_{-2} = \frac{4\pi^2 M}{W_{\max}^2} + \frac{\varepsilon\lambda W_{\max}^2}{4\pi^2} \quad (30)$$

Section ‘‘Some Typical Numbers’’ includes estimates of the numbers in G_2 and G_{-2} .

Finding $\varepsilon\lambda$

Because of equation (26), it makes sense to choose $\varepsilon\lambda$ so that $G_{-2} \approx G_2$. Therefore, set

$$\frac{4\pi^2 M}{W_{\max}^2} + \frac{\varepsilon\lambda W_{\max}^2}{4\pi^2} \approx M \left(E_{\max}^2 + \frac{4\pi^2}{W_{\min}^2} \right) + \frac{\varepsilon\lambda}{E_{\max}^2 + 4\pi^2/W_{\min}^2}$$

That is

$$\begin{aligned} \varepsilon\lambda & \left(\frac{W_{\max}^2}{4\pi^2} - \frac{W_{\min}^2}{E_{\max}^2 W_{\min}^2 + 4\pi^2} \right) \\ & \approx M \left(E_{\max}^2 + 4\pi^2 \left[\frac{1}{W_{\min}^2} - \frac{1}{W_{\max}^2} \right] \right) \end{aligned}$$

Because $W_{\max} \gg W_{\min}$, this simplifies to

$$\varepsilon\lambda \approx \frac{4\pi^2 M}{W_{\max}^2} \left[E_{\max}^2 + \frac{4\pi^2}{W_{\min}^2} \right] \quad (31)$$

Finding ε

In equation (24) it would be convenient to set $\kappa^2 = 4m\varepsilon$, but it is derived from equation (13), and the quantities in equation (13) involve estimation. It is therefore more realistic to write

$$\kappa^2 = 4m\varepsilon - \nu \quad (32)$$

for a small value ν . Equation (24) then becomes

$$2mG^2 + \nu G \cos 2\mu - \varepsilon(2m\varepsilon + 8mM\lambda \sin^2 2\mu - \nu) + (4m\varepsilon - \nu)^2(1 - H^4)/8m < 0 \quad (33)$$

The roots G_{\pm} of equation (33) are given by

$$4mG_{\pm} = -\nu \cos 2\mu \pm \sqrt{\nu^2 \cos^2 2\mu + 8m\varepsilon(2m\varepsilon + 8mM\lambda \sin^2 2\mu - \nu) - (4m\varepsilon - \nu)^2(1 - H^4)}$$

Therefore

$$4mG_{\pm} = -\nu \cos 2\mu \pm \sqrt{\nu^2 \cos^2 2\mu + 8m\varepsilon H^4(2m\varepsilon - \nu) + 64m^2 M\varepsilon\lambda \sin^2 2\mu - \nu^2(1 - H^4)} \quad (34)$$

It is clear from equation (34) that G_{-} is negative and so satisfies $G_{-} \leq G_{\min} = 2\sqrt{M\varepsilon\lambda}$ as required by equation (25). It remains to show that we can choose ε and λ so that $G_{+} \geq G_{\max}$, that is

$$(4mG_{\max} + \nu)^2 \leq 8m\varepsilon H^4(2m\varepsilon - \nu) + 64m^2 M\varepsilon\lambda \sin^2 2\mu \quad (35)$$

The second-order small terms in ν^2 have been discarded and a small element $\nu/4m$ added to G_{\max} to cover the uncertainty of the sign of $\nu \cos 2\mu$. Write $G'_{\max} = G_{\max} + \nu/4m$ in inequality (35):

$$G_{\max}^{\prime 2} \leq H^4 \left(\varepsilon^2 - \frac{\nu\varepsilon}{2m} \right) + 4M\varepsilon\lambda \sin^2 2\mu$$

Since the term in $\sin^2 2\mu$ may vary between zero and one, the inequality is satisfied without this term provided that $\varepsilon\lambda > 0$. We therefore arrive at a quadratic equation in ε as follows.

$$\varepsilon^2 - \frac{\nu\varepsilon}{2m} - \frac{G_{\max}^{\prime 2}}{H^4} = 0$$

The solution is

$$\varepsilon = \frac{\nu}{4m} \pm \frac{1}{2} \sqrt{\frac{\nu^2}{4m^2} + 4 \frac{G_{\max}^{\prime 2}}{H^4}} \quad (36)$$

In fact, the requirement is more simply met (and slightly over-fulfilled) by setting

$$\varepsilon = \frac{G_{\max}}{H^2} + \frac{\nu}{m} \quad (37)$$

Because $G_{-2} \approx G_2$, we can take $G_{\max} \approx G_{-2}$. Then from equations (30) and (31)

$$G_{\max} \approx \frac{4\pi^2 M}{W_{\max}^2} + \frac{W_{\max}^2}{4\pi^2} \left\{ \frac{4\pi^2 M}{W_{\max}^2} \left[E_{\max}^2 + \frac{4\pi^2}{W_{\min}^2} \right] \right\}$$

Therefore

$$\begin{aligned} G_{\max} &\approx M \left[E_{\max}^2 + 4\pi^2 \left(\frac{1}{W_{\max}^2} + \frac{1}{W_{\min}^2} \right) \right] \\ &\approx M \left[E_{\max}^2 + \frac{4\pi^2}{W_{\min}^2} \right] \end{aligned}$$

Hence and by equation (37)

$$\varepsilon \approx \frac{M}{H^2} \left[E_{\max}^2 + \frac{4\pi^2}{W_{\min}^2} \right] + \frac{\nu}{m} \quad (38)$$

This completes the derivation of values of ε and λ that ensure convergence of the solution for all physically possible values of ϕ and ψ .

DESIGN OF ACTUATORS

The purpose of the actuators is to vary the bending compensation B_a according to the equations. The options are either to use servo or solenoid mechanisms external to a pair of tubes or to rely on somewhat more sophisticated electronics to adjust the electromagnets on each bolt.

An external mechanism has the drawback that it adds to the weight and to the bulk. The bulk will increase the wind resistance and will make the design of the bearer for launch vehicles more complicated. However, it has the advantage that it does not have to be provided along the whole length of the space cable but only where wind may be encountered.

Using the bolts' own electronics reduces the number of moving parts and so simplifies the mechanical design. The electromagnets are already present on the bolts to enable them to perform their primary function of levitating the tube. The bending compensation is an additional factor that amounts to providing differential forces between the middle and the ends of each bolt. In view of the reliability and cheapness of electronics today, this is the preferred design.

Wind sensors are required outside the tubes. In the preferred design, they communicate the needed information to the bolts on the inside as they come past. Electronics associated with the wind sensors performs the calculations necessary to determine the value of B_a in equations (7) and (13) and passes this value to each bolt. A very simple communication mechanism is proposed: the frequency of an oscillator in the tube

represents the value of B_a and is passed to the bolts by inductive or capacitive coupling.

Above the windy zone, occasional correction in position can be performed if the bolts are used as actuators. A few position sensors, probably using GPS, can be used to check long-term stability.

SUPPORT AT THE SURFACE STATIONS

Previous calculations¹ for the support structures at the surface stations are still applicable with the introduction of active curvature control. In the design of Version 1, the supports consist of a 400-metre tunnel, a 200 metre gantry and over 300 metres of support tubes. The support tubes are steerable so that they can vary both the angle of inclination of the main tubes and their lateral deflection. The cautious assumptions made were that there might be a deflection of up to 14° on top of a substantial angle of inclination while the space cable is being erected.

The support tubes use the same technology of bolt levitation as the main tubes but are much smaller. One advantage over the previous design assumptions is that the combination of wind force and active curvature control cause the space cable to move with the wind; the support tubes do not have to provide the power to deflect it. Rather, there is in principle a source of power that could be captured at the surface station as the cable moves in response to changes in wind. Whether this is a worthwhile source of power is moot, as it depends on gusts and changes, but at least it need not consume much power.

SOME TYPICAL NUMBERS

To obtain a value for ε from equation (38), we need values for M , H , E_{\max} and W_{\min} . Now $M = m_b V^2 - T$. Previous work on Version 1¹ has taken the average bolt mass m_b as 2 kg/metre and the velocity V as 2.5 km/sec. As the tension T is low at these altitudes, this gives $M \approx 1.25 \times 10^7$.

In the preferred design, the bolts perform the role of actuators. The lowest wavelength they can accommodate is twice the length of a bolt. Below this wavelength, the stiffness of the bolt itself prevents waves, as explained before.⁴ A bolt is 1 metre long, and so $W_{\min} = 2$.

Example for finding E_{\max}

To find E_{\max} consider an extreme example in which there is initially no wind until a crosswind suddenly commences between the heights of 6 km and 12 km. Suppose that this wind operates from one side to another and is uniform along this length.

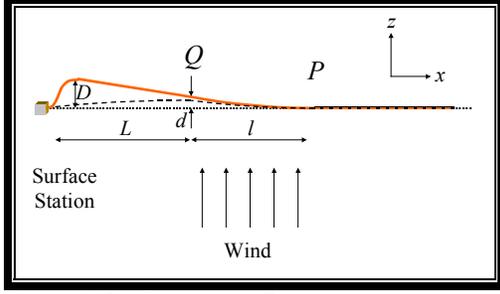


Figure 4 Displacement in Progress

Initially the cable is in a vertical plane at an angle to the horizontal of about 60° . The length l of the windy part of the cable is $6/\sin 60^\circ$, or about 7 km. The lower part where there is no wind has approximately the same length ($L=l$). After the onset of the wind, the lateral displacement moves from zero to a curve that exactly opposes the wind. As illustrated in Figure 4, this causes the lower part of the cable to line up at an angle, thus transmitting the wind force to the ground station.

The curvature to oppose the wind is given by $c_r = F/M$ from equation (6). If the wind force is at the predicted maximum of 50 Newtons/metre, the radius of curvature $r_r = 1/c_r$ is about 5×10^5 metres. The angle θ subtended by this curved section at the centre of radius is approximately $\theta = \frac{l}{r_r} \approx \frac{7 \times 10^3}{5 \times 10^5} = 0.014$ radians or about $1\frac{1}{2}^\circ$. Then the lateral displacement at point Q in Figure 4 is given by $d = l \sin \frac{\theta}{2} \approx 7 \times 10^3 \times 0.014/2 = 49$ metres. The displacement at the ground is $D = d + L \sin \theta$. This comes to $D \approx 49 + 7 \times 10^3 \times 0.014 = 147$ metres.

Because this is an extreme example, we can take this value of D as D_{\max} in equation (29) with $x_R = l + L$, giving $E_{\max} = 3.6 \times 10^{-4}$.

Values of ϵ , κ and $\epsilon\lambda$

Having values of $M \approx 1.25 \times 10^7$, $E_{\max} \approx 3.6 \times 10^{-4}$ and $W_{\min} = 2$, take $H=0.9$ in equation (38) to get: $\epsilon \approx 1.5 \times 10^8$.

Previous work has taken the average mass m (bolt plus tube) as 3 kg/metre, making $\kappa \approx \sqrt{4m\epsilon} \approx 4.2 \times 10^4$ from equation (32). Thus and from equation (22), since $z' = \gamma e^{\psi + \kappa x}$, the temporal convergence of the cable is governed by a negative exponential term $\psi < -70$. This will ensure quick convergence.

To obtain $\epsilon\lambda$ from equation (31), the value of W_{\max} is needed. Since the space cable is anchored at both ends, the maximum wavelength is twice the cable length,

making W_{\max} about 700 km in Version 2 and half that in Version 1. Taking the greater value gives $\epsilon\lambda \approx 0.1$.

Forces in an Actuator

As the bolts are being used both for levitation and as curvature actuators, it is useful to consider the forces needed. In previous work¹ the maximum magnetic force per bolt for levitation is given as 570 Newtons and is provided by three pairs of electromagnets inclined at angles of 45° . Each magnet therefore needs to exert forces up to $570/3\sqrt{2}$ Newtons, i.e., up to 135 Newtons. The actuators exert a bending compensation B_a by altering the forces in these magnets, but they still need to have the same sum.

Let the end forces and middle force be F_e and F_m respectively and the distance between them be l_a , which is half the bolt length. Then B_a is the discrete equivalent of the second partial derivative of the force with respect to x .

$$B_a = \frac{(F_e - F_m)/l_a - (F_m - F_e)/l_a}{l_a} = 2 \frac{F_e - F_m}{l_a^2}$$

In the example, the wind force F is constant after its sudden start, and so equation (7) implies $B_a = B_c$ in this case. Equation (13) then gives values for B_a . In the example, the curvature $c_r = 2 \times 10^{-6}$ and so the $\epsilon(c - c_r)$ term in equation (13) is approximately 300, which represents a maximum. The damping term $\kappa \frac{\partial}{\partial t}(c - c_r)$ will tend to oppose $\epsilon(c - c_r)$, but the value of 300 remains the overall maximum. The contribution from the term $\epsilon\lambda z'$ between points P and Q in Figure 4 is up to 5 and can reach a value of 15 near the surface station.

Taking $B_a \approx 300$ and $l_a = 0.5$, the force difference $F_e - F_m = \frac{1}{2} l_a^2 B_a$ is about 38 Newtons. This variation amounts to a modest ± 25 Newtons on the previously calculated maximum of 135 Newtons per electromagnet, although even this requirement can be relaxed, as strong winds occur only at low altitudes where the required levitation force is much lower.

Discussion of the Example

Referring to equation (4) and Figure 4, the bending stress B due to wind is non-zero in this example only at points P and Q where the wind changes. At these two points the curvature derivative $\frac{\partial^2 c}{\partial x^2}$ is non zero, and it matches the value of B at that point. Since $\frac{\partial^2 c}{\partial t^2} = 0$ in the stable position, only very small bending stresses B_a have to be applied by the curvature

actuators anywhere to maintain stability until the wind changes.

At the initial onset of the crosswind, it is necessary for the curvature actuators to adjust according to the equations so as to ensure a smooth transition to the new stable position. Without active curvature control, the section PQ of the cable would accelerate *en bloc* with the wind.

At point P , the bending stress B is caused by the presence of the wind force below P at 50 Newtons/metre but zero above P . Let the actuator length be l_a . Then the bending stress is averaged over l_a . At point P :

$$B = \frac{l_a F}{l_a^2} = \frac{F}{l_a} \quad (39)$$

As a working assumption, the actuators are taken to be 1 metre long, and so the bending stress is averaged over a metre. The value is thus 50 Newtons/metre². Initially, the curvature actuators transmit this bending stress along the cable below P until the cable reaches the stable position z_r and curvature c_r . None of the bending stress is transmitted above P , since the cable above P is already in the stable position. Below and near to P the cable soon reaches the stable position, and the bending stress drops to zero. This can be seen from equations (4), (7) and (13), which may be combined as follows.

$$B_a = -\varepsilon(c - c_r) - \varepsilon\lambda(z - z_r) - \kappa \frac{\partial}{\partial t}(c - c_r) + M \frac{\partial^2 c}{\partial x^2} - B + \frac{m}{M} \frac{\partial^2 F}{\partial t^2} \quad (40)$$

Below and near to P the dominant term initially is $\varepsilon(c - c_r)$, since the cable is already close to the target displacement z_r . As the cable moves, the curvature c reaches its target value first near P and later lower down. Thus there is a variation in the curvature due to the fact that lower parts of the cable have further to go to reach their target positions and curvatures. This is reflected in a significant value for the term $M \frac{\partial^2 c}{\partial x^2}$.

At P , this term eventually balances the bending stress B , and the other terms drop to zero. Below P , as the cable approaches the stable position, all the terms drop to zero (or close to it), including $M \frac{\partial^2 c}{\partial x^2}$. Nearer point Q , the term $\varepsilon\lambda(z - z_r)$ assumes greater significance during the movement, as the distance for the cable to travel is up to 49 metres.

At Q , there is an external bending stress B . Its initial effect is to cause the cable to curve in the direction that helps it to reach its target position z_r . The damping term $\kappa \frac{\partial}{\partial t}(c - c_r)$ partly resists this change to ensure smooth movement. There is a fairly complicated interplay between the other terms until eventu-

ally the stable position is reached. Below Q , the dominant term is $\varepsilon\lambda(z - z_r)$, which causes the cable to form a bow shape (Figure 4). This is related to the intermediate curvature referred to as c_c and defined in equation (10). It effectively spreads the effect of the bending stress at Q so as to move the whole of the cable below Q towards the required position z_r .

CONCLUSION

The solution to the problem of ensuring lateral stability in the presence of cross winds has been shown to be possible using electromagnetic forces controlled electronically. It remains to verify this solution using computer simulation and eventually working models. In the meantime, there is further mathematical work to do to adapt this solution to the slightly more complicated case of stability in the plane of the space cable and also to examine the effect of winds longitudinally along the tubes.

In view of the common use of electronics for fly-by-wire systems in commercial aircraft, the reliability of electronic controls is now well established. They are generally more efficient than comparable mechanical solutions. Certainly, the solution presented here is much lighter in weight than that presented before⁴ and is likely to be more cost effective.

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