THE SPACE CABLE: CAPABILITY AND STABILITY

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The space cable is an electromagnetic launcher at high altitude. It is supported by fast-moving projectiles, called bolts, inside evacuated tubes using magnetic levitation. It can replace the function of a first-stage rocket and is suitable as a platform for astronomy and other science. A larger configuration is capable of launching manned vehicles directly to orbit. There is a lightweight solution to the challenge of lateral stability that uses electronic controls in the bolts and tubes.

Keywords: Space cable, dynamic structures, electromagnetic launcher

1. INTRODUCTION

The space cable [1] consists of several evacuated tubes rising tens of kilometres above the Earth's surface and covering a range of 150 km or more over the ground or sea. It is a development of the launch loop [2] on a smaller scale. The tubes are supported by fast travelling projectiles inside them, called bolts, using magnetic levitation. The levitation mainly uses permanent magnets to minimise power consumption.

The space cable can be used for launching vehicles to space, including manned vehicles. Like the space elevator [3], it can support gentle rides into space for tourists at speeds similar to a train; it can provide a permanent platform for scientific installations, such as space telescopes, to which it allows economical access for servicing and repairs. Unlike the space elevator, the space cable is built from the ground or sea up and does not need parts of it to be lowered from geosynchronous orbit. The space elevator requires a new strong material to be developed, whereas the space cable uses known materials and adaptations of known technologies.

The space cable's travelling bolts transmit the necessary power and thrust for launching vehicles into space using a combination of linear electric generators and motors. In some respects, it is comparable to an electromagnetic launch ramp [4], which uses linear electric motors to provide thrust. There are two main forms of electromagnetic launch ramp:

i. A short ramp can accelerate a rocket along the ground to a few hundred miles an hour, thus saving a useful amount of fuel at launch time. These ramps have been demonstrated and are suitable for manned or unmanned craft.

ii. There are proposals for launchers that accelerate unmanned vehicles above Earth escape velocity. One such launcher consists of a tube 2 km in length from which the vehicle emerges. Acceleration may be several hundred *g*, after which the vehicle faces heating and energy loss due to atmospheric drag. With sufficient initial speed, it appears possible for vehicles of one or two tons to reach high orbit.

The space cable is levitated so as to rise above most of the atmosphere and avoid significant air drag.

To illustrate the range of possibilities, three versions (Fig. 1) are described here:

1. Version 1 [1] rises to 50 km and covers a range of 150 km over the Earth's surface. It can accelerate a 90 tonne vehicle to 1.6 km/sec, thus performing the role of a first-stage rocket.
2. Version 2 [5] rises to 140 km and covers a range of 175 km. 140 km is chosen as an upper height limit because, above this level, the weight and associated costs increase much faster. The higher elevation increases the space cable's usefulness for communications and science. Vehicles could transfer from here to an orbiting tether such as the skyhook [6]. The costs are about triple those of version 1.
3. Version 3 also rises to 140 km but covers a range of 900 km. This version can launch manned vehicles directly to near Earth orbit without the need for rockets. However, the cost is 10 times that of version 1. It represents a maximum, beyond which a different design is needed, such as the launch loop.

A key issue is stability, especially in the presence of winds, including the jet stream. The external diameter of a tube is of the order of 5 to 10 cm. Ten tubes comprise a typical system to provide reliability through redundancy. There is virtually no inherent strength to control bending and so a method known as active curvature control [7] has been devised. This can achieve lateral stability using the capabilities of the bolts. The bolts use a combination of permanent and electromagnets to maintain a safe spacing from the inner walls of the tubes while sustaining the tubes' weight. Active curvature control uses the same magnets to adjust the tubes' curvature so that they can counteract winds and correct their position.

The present paper gives a more complete treatment of active curvature control than previously published, including the case where the bolts' spacing and speed vary between tubes, as

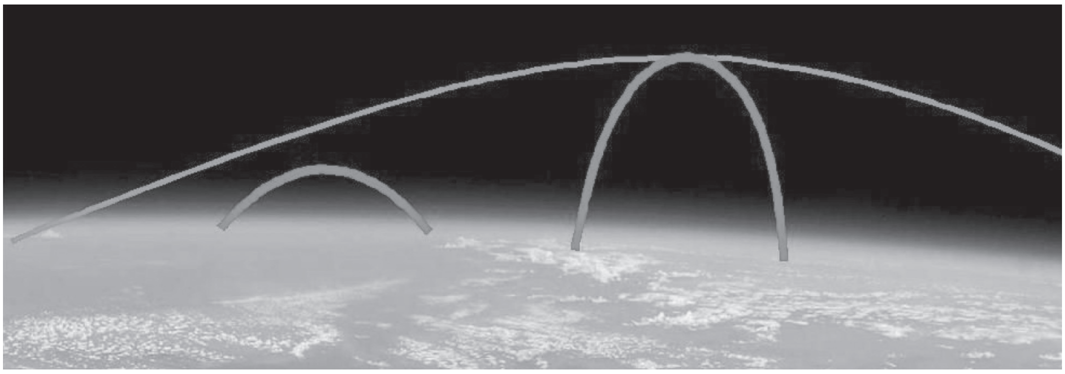

Fig. 1 Comparison of three versions.

happens when launching a spacecraft. Without this active control the displacement at the top of the cable could amount to 6 or 7 km.

2. SUMMARY OF DESIGN

The cable is supported at two surface stations, either on land or sea. At each surface station, there is an accelerator/decelerator for starting or stopping bolts. There is a tear-shaped circuit, called the ambit, (Fig. 2) to turn bolts around so that they return to the cable. There is an arrangement of tunnels and gantry, collectively termed the ramp, that cause a bolt to rise to the correct angle so that it travels through one of the tubes to the other surface station and back again indefinitely. The ramp can apply some acceleration or deceleration to the bolts. When a launch is imminent, it accelerates the bolts so that their extra energy can be transferred to the vehicle. After a launch, the ramp has to restore an even flow to the bolts.

As there are several pairs of tubes, it is possible to use the decelerator to quiesce one pair and take it down for servicing while the other tubes remain in service. Crawlers are used to drag one pair of tubes along the others, both for maintenance and during initial construction.

The length of the accelerator/decelerator and the size of the ambit are important factors in the cost; the ambit requires superconducting magnets and the accelerator/decelerator uses powerful electromagnets. The size of these installations is proportional to a bolt's mass and the square of its speed.

The bolts' kinetic energy has to be sufficient to attain the required height. In addition, the bolts support the tube in which they travel, plus any payload. 140 km is a satisfactory upper height limit to avoid space debris; at this height debris reenters the atmosphere and burns up. Smaller or intermediate-sized versions of the space cable are feasible: for instance, a 20 km high cable could support a stratospheric wind farm and, in the still air above that, a set of thin solar panels permanently above cloud. The bolts can be used both to transmit and store energy.

The bolts' reliability is a critical factor in the design due to the consequences of a bolt coming into contact with the tube wall. Techniques used in fly-by-wire and other safety-critical electronic systems need to be adapted and applied to ensure this reliability. One suggested approach is to carry out diagnostic tests on each bolt when it enters the ramp so that the accelerator/decelerator can remove and replace a suspect bolt.

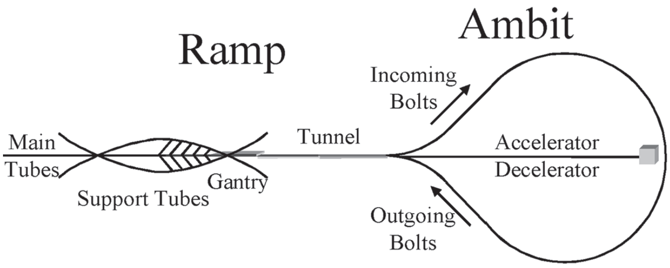

Fig. 2 Layout of a surface station.

2.1 Dynamics of Launching

The bolts are given about 20% extra kinetic energy to assist a launch. The extra energy causes an increase in the cable's tension, which is manageable within limits. A vehicle is propelled by electromagnetic drag from the passing bolts and tends to catch up with them. Each bolt gives it an additional impetus as it passes but, as the vehicle catches up, fewer bolts pass it and their relative velocity is small, so the impetus available decreases. This electromagnetic drag creates surplus energy; the arrangement is like a linear electric generator with a slipping stator.

The vehicle is propelled by a bearer to which it is attached for launching. The bearer contains the linear electric motors and generators together with systems for magnetic levitation. When the vehicle and bearer reach the target velocity, the bearer releases the vehicle. The bearer remains magnetically attached to the tubes and gradually slows down, giving up energy to the bolts in a form of electro-regenerative braking as it travels to the other surface station.

The bearer is 150 metres long. Over five pairs of tubes, it obtains a thrust of 4.5 MN. [8] The modelling tool *Finite Element Method Magnetics (FEMM) 4.2* was used to obtain an estimate of the bearer's mass. The electromagnets sum to about 7 tonnes, to which must be added thrust-bearing members, cooling equipment and levitation support, leading to an overall estimate of 15 tonnes.

3. ILLUSTRATIVE VERSIONS AND EXAMPLES

The trajectories of the three versions of the space cable (Fig. 1) are calculated using the equations given in [1]. For a given height and length, together with the bolt mass and spacing, the required bolt speed and direction of inclination are calculated numerically using a computer model. The bolt mass and spacing chosen for each version is designed to minimise the levitation force required per bolt.

In all versions, a basic tube mass of 2 kg per metre is assumed, which includes 1 kg of NIB magnets per metre. Each tube has an additional variable mass of Kevlar® structure, which increases with height; it is needed to support the lower parts of the tube.

3.1 Version 1 Example

The 7 kg bolts are spaced 5 metres apart. When the space cable is idle, the bolt speed at the surface is 2 km/sec. During a launch, the surface station from which the vehicle starts increases this to 2.2 km/sec. The bearer removes this excess speed as it extracts momentum from the bolts. A combined mass, including the bearer, of 90 tonnes can be accelerated to 1.6 km/sec over a distance of 65 km [8]. The half length, or runway, (i.e., distance from the surface to the top) of version 1 is 92 km. This gives room to accelerate 105 tonnes to this speed over 80 km. As the bearer's mass is 15 tonnes, this allows for a 90 tonne vehicle.

3.2 Version 2 Example

The 7 kg bolts are spaced 2.5 metres apart. The bolt speed is 2.8 km/sec, increasing to 2.95 km/sec for a launch. Compared to version 1, the higher bolt speed permits a higher launch speed. A 105-tonne mass reaches 2.4 km/sec over 60 km. This launches a 90-tonne vehicle, allowing for the 15-tonne bearer. The runway notionally available in version 2 is 180 km, although the lower parts of this are rather steep due to the shape of the trajectory (Fig. 1). Consequently, it is desirable to use the higher elevations.

3.3 Version 3 Example

The 5 kg bolts are spaced 4.5 metres apart. The bolt speed is 10.4 km/sec, increasing to 10.9 km/sec for a launch. The available runway is 525 km. This is long enough to launch a manned vehicle directly to orbit.

For launching a manned vehicle, acceleration is limited to $a \approx 6g$, or about 60 metres/sec^2. The distance to reach $v = 7.9$ km/sec in time t is given by the well-known formula

$$D = \frac{1}{2}at^2 = \frac{v^2}{2a} \approx \frac{(7.9\times10^3)^2}{2\times60} \approx 520 \text{ km}$$

Combined with a contribution of 0.3 km/sec from the Earth's rotation if launching towards the east, this is enough to reach a near Earth orbit.

The mass of the vehicle and bearer is found from the ratio of thrust to acceleration. Since the thrust is 4.5 x 10^6 Newtons, the mass is 4.5 x 10^6/60 ≈ 75 x 10^4 kg or 75 tonnes. Allowing for the 15 tonne bearer, the mass launched is 60 tonnes.

4. ACTIVE CURVATURE CONTROL

Any bend or bulge in a tube will cause turning of the bolts travelling inside. This turning creates a centrifugal force. Uncontrolled, the centrifugal force will increase the bend or bulge indefinitely, leading to catastrophic instability. However, careful control allows the centrifugal force to be exploited so as to counteract external forces, mainly wind.

At high altitudes (above 12-15 km), the air has a very low density and consequently the effect of any turbulence is small and there are no other lateral forces. However, there are strong cross winds lower down so while in calm air the space cable has no lateral bending, in reality it will need to allow for wind loading. The proposed technique to handle these loads is to measure the wind speed using a form of anemometer and adjust the bending so as to counteract the wind exactly, as in Fig. 3. The bending is designed to cause a change in the gradient of the space cable below the region of wind (i.e., nearer the surface station) but not higher up. In this way, the lateral force is transmitted down to the surface station and not upwards. At the surface station, this force can be absorbed by means of an opposite bend, and the higher part of the space cable remains steady.

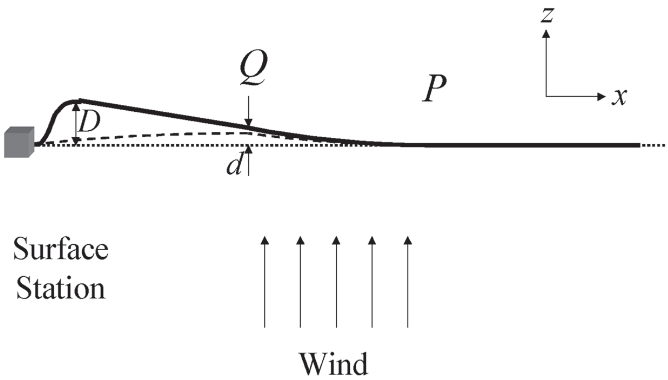

Fig. 3 Displacement in progress.

To adjust the curvature over a short length of tube, the bolts' own electromagnets are used. Figure 4 shows a pair of bolts. The permanent magnets run the length of each tube. The electromagnets in the bolts that are used for curvature adjustment are already used for levitating the tubes, as illustrated. When the wind changes, it causes some bending. The purpose of the adjustments is to limit and direct that bending.

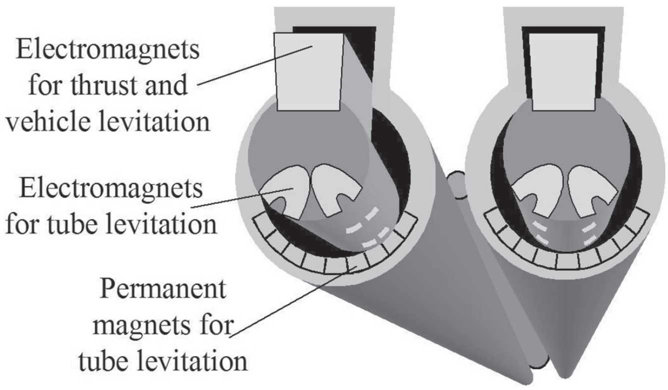

Fig. 4 Cross section of tubes with bolts.

There are three pairs of electromagnets in each bolt. This means that they can apply a curvature correction, or bending compensation, which is based on the discrete equivalent of second-order partial differentiation in which a pair of electromagnets in the middle apply a force different from that applied at each end. A bolt is a metre long, and so the tubes must be flexible enough to move in response to these forces. A tube consists of jointed sections each 30 cm long. Then each of the three pairs of electromagnets in the bolt is applying a force to a different section of the tube.

The wind sensors outside the tubes communicate the needed information to the bolts on the inside as they come past. Electronics associated with the wind sensors performs the calculations necessary to determine the bending compensation and passes this value to each bolt. A very simple communication mechanism is proposed: the frequency of an oscillator in the tube represents the magnitude of bending compensation, which is passed to the bolts by inductive or capacitative coupling.

Above the windy zone, occasional corrections in position can be performed. A few position sensors, probably using GPS, can be used to check long-term stability.

The details of the bending compensation are in the appendix. As shown there, the following two inequalities must be satisfied to achieve stability.

$$\varepsilon > N\frac{2\pi^2}{W_S{}^2}+(1-H^2)\frac{\kappa^2}{2m} \qquad (1)$$

$$\kappa > 0 \qquad (2)$$

Here N is related to wind shear (see Section 4.2), W_S is the shortest wavelength of standing waves in the tubes, and m is the average per unit distance of the combined mass of a pair of tubes and the bolts travelling inside them. H is a constant with $0 < H < 1$ that governs, in conjunction with κ, the speed at which the space cable moves to a new stable position whenever the wind changes. The constants ε and κ are part of the bending compensation exerted by the bolts on the tubes. Section 4.2 includes estimates of these numbers.

4.1 Lateral Displacement due to Wind

Consider an extreme example in which there is initially no wind until a crosswind suddenly commences between the heights of 6 km and 12 km. Suppose that this wind operates from one side to another and is uniform along this length. This example has been verified using a finite-element computer model.

Initially the cable is in a vertical plane at an angle to the horizontal of about 60°. In Fig. 3, the windy part between P and Q has a length of 6/sin 60°, or about 7 km. The lower part (to the left of Q) where there is no wind has approximately the same length. After the onset of the wind, the lateral displacement moves from zero to a curve that exactly opposes the wind. As illustrated in Fig. 3, this causes the lower part of the cable to line up at an angle, thus transmitting the wind force to the ground station.

The curvature required to oppose a wind force F is given by $c_r = 2F/M$ (equation (A.9) in the appendix), where

$$M = m_a V_a{}^2 + m_d V_d{}^2 - 2T$$

In version 1, the average bolt masses m_a and m_d are 1.4 kg/metre; the ascending and descending velocities V_a and V_d are 2 km/sec. The tension T in each tube is low. Hence $M \approx 1.1 \times 10^7$. If the wind force is at the predicted maximum of 50 Newtons/metre [1], the radius of curvature $1/c_r$ is about 1.1×10^5 metres. Some trigonometry gives the displacement at the ground as $D \approx 670$ metres. In version 2, $D \approx 160$ metres; in version 3, $D \approx 30$ metres.

Previous calculations [1] for the size of the deflection support tubes at the surface stations need to be increased from about 300 metres to 1000 metres to accommodate the 670 metre deflection in version 1. 300 metres will suffice for versions 2 and 3. Figure 5 shows a side view of version 1 with the Eiffel Tower included for comparison.

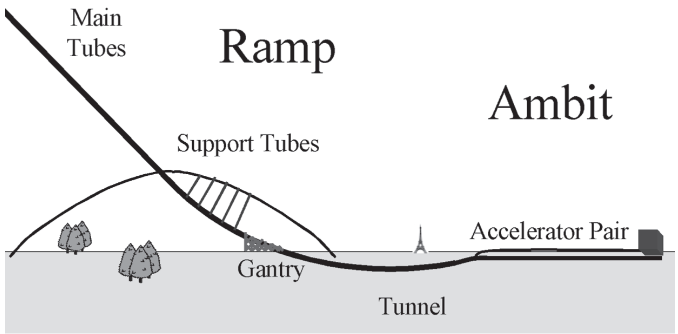

Fig. 5 Support structure at surface station.

4.2 Wind Shear

The value N in inequality (1) involves estimating

$$\frac{\partial^2 c_r}{\partial x^2}$$

where $Mc_r = 2F$. (See equations (A.8), (A.9) and (A.32) in the appendix.) If the wind velocity is v_w metres/sec, previous work [1] has shown that $F \approx v_w/2.2$. The wind shear is

$$\frac{\partial v_w}{\partial x}$$

and has been researched because of its impact on aircraft flight safety. A NASA study [9] showed a maximum figure of about 0.03 per second in the presence of strong jet streams winds. This translates to a value of

$$\frac{\partial c_r}{\partial x} \approx \frac{0.03}{2.2}\frac{2}{1.1\times10^7} \approx 2.5\times10^{-9}$$

Derivatives of wind shear do not appear to have been researched but are likely to be even smaller.

The difficulty is in measuring the derivative of wind shear. This uncertainty can be transferred to a measuring error in N. In principle, an overestimate could ensure that N is always negative, which would allow a very small value for ε. However, if the wind shear derivative is small, it may not even be possible to get its sign correct. Therefore an estimating error must be allowed. If this is 10%, it gives a value for N of about $M/10$. This is $N \approx 1.1 \times 10^6$ in version 1, $N \approx 4.4 \times 10^6$ in version 2 and $N \approx 2.4 \times 10^7$ in version 3.

4.3 Some Typical Numbers

The dominant term in inequality (1) contains N and W_S. With tube sections 30 cm long, the shortest wavelength in a tube is $W_S = 1.8$ metres. With the numbers in Section 4.2,

$$\varepsilon \approx N(2\pi^2/W_S{}^2) \approx 1.2\times10^7$$

in version 1, $\varepsilon \approx 4.8 \times 10^7$ in version 2 and $\varepsilon \approx 2.6 \times 10^8$ in version 3.

The dominant term in the bending compensation B_{ea} is $\varepsilon(c_r - c)$ (see equation (A.8)). The maximum value is about 110 for all three versions. However, this is an average per metre. In version 2, the bolts are a metre long and 5 metres apart, and so the value of the curvature correction term per bolt is 550.

Choosing $\kappa = 10$ and $H = 0.9$ has no appreciable effect on ε.

The temporal convergence is governed by $e^{\psi t + \varphi x}$ in equation (A.22) with $\psi \approx -2(1 - H)\kappa \approx -2$ from equation (A.33). This causes a reduction to one hundredth of the amplitude of any oscillation in time $t = \frac{1}{2} \ln 100 \approx 2.3$ seconds.

4.4 Forces in a Bolt

As the bolts are being used both for levitation and as curvature actuators, it is useful to consider the forces needed. The maximum levitation force per bolt in version 1 is 400 Newtons. In versions 2 and 3, the force is 1.2 kN. It is provided by three pairs of electromagnets inclined at angles of 45°. Each magnet therefore needs to exert forces up to 400/(6cos45°) or 1200/(6cos45°) Newtons, i.e., 95 or 285 Newtons. The bolts exert the bending compensation B_{ea} by altering the forces in these same magnets.

Let the end forces and middle force be F_E and F_M respectively and the distance between them be l_a, which is half the bolt length. Then B_{ea} is the discrete equivalent of the second partial derivative of the force with respect to x.

$$B_{ea} = \frac{(F_E - F_M)/l_a - (F_M - F_E)/l_a}{l_a} = 2\frac{F_E - F_M}{l_a^2}$$

In Section 4.2, the dominant term in B_{ea} has a maximum of 550. Taking $l_a = 0.5$, the force difference $F_E - F_M = \frac{1}{2} l_a^2 B_{ea}$ is 69 Newtons. This variation is applied while keeping the overall force $2F_E + F_M$ constant. Hence the worst case is to take $\frac{2}{3}$ of the increase at the centre of the bolt, where it is shared by two electromagnets inclined at 45°. This gives a maximum increase of 33 Newtons on the levitation force. This is well within the maxima calculated above of 95 or 285 Newtons per electromagnet, which are needed only near the top of the space cable, well away from the windy region where stabilisation is needed.

4.5 Energy Conservation

The bolts are designed to use very little energy for their main task of levitating the tube in which they travel. Although they contain electromagnets, these operate largely passively, relying on the magnetic attraction between their ferrite cores and the permanent magnets in the tube. The electromagnets use very small currents to maintain stability and keep a constant distance. Lateral forces, however, are unpredictable and will require appreciable expenditure of energy by the electromagnets.

The strongest lateral forces will be the centripetal forces exerted by the bolts themselves to make them turn round bends in the tubes. These directly counteract the wind, and so the maximum is simply the wind force per bolt. For a bolt spacing of 5 metres and wind of 50 Newtons/metre, the maximum force per bolt is 250 Newtons, which is 250/(6cos45°) per electromagnet or about 60 Newtons. This is within their maxima.

It may be possible to devise a scheme in which the bolts move nearer to or further from the permanent magnets so as to vary the force more economically. To make the bending compensation more economical, it is necessary to improve the accuracy of the wind-shear measurements.

Thin-film solar panels could be suspended from the higher reaches of the space cable, above the regions of cloud and wind, to generate during the hours of daylight the electric power lost. It may even be worthwhile to install enough panels to generate a surplus of electricity for sale, since the sunlight would be uninterrupted during the day and stronger than on the ground.

5. COST ESTIMATES

Retail prices of the main commodities in US dollars are used to allow for assembly and other costs.

In each tube, about 1 kg per metre of NIB magnets is needed in versions 2 and 3. Version 1 only requires about a third of this. The cost is about \$250 per kg. The tubes also need ferrites, expansion joints and vacuum-tight materials, which are estimated at \$50 per metre. The main supporting material is Kevlar, which costs about \$90 per kg. The amount of Kevlar needed is estimated numerically using the basic equation [1].

It is assumed that all three versions use the same bolt design. Versions 2 and 3 need about three times the levitation force needed by version 1; this is reflected in a lower mass estimate for the NIB magnets in the tubes in version 1. A bolt has six electromagnets for levitation of the tube. The size and specifications suggest a retail price of \$25 each. Ten larger electromagnets are needed for thrust and levitation of vehicles, estimated at \$28 each. In addition, each bolt has electronics and structural elements costing another \$150, giving a total per bolt of \$580.

At the surface stations, superconducting magnets are housed in tunnels or trenches. At CERN in Geneva, complete refurbishment of 27 km of tunnel with rather more sophisticated facilities cost about \$2.5 billion, about \$93 million per km. This figure is used for estimates of the surface station's cost. The ambit radius is given by $m_b V^2/F_A$, where the force F_A exerted by the superconducting magnets in the ambit is 190 kN. The bolt mass m_b and maximum speed V vary by version.

Each version is assumed to have a bearer and two tourist vehicles. These are comparable in complexity to an airliner; allowing \$200 million each, the total is \$600 million.

5.1 Version 1

The tube length is 188 km. Taking the NIB cost as \$85 per metre, the cost of NIB and other materials comes to \$135 per metre. Over 10 tubes this is \$250 million. The mass of Kevlar® per tube is calculated as 410,000 kg, giving a total cost over 10 tubes of \$370 million. There are 46 support tubes at each surface station, costing an estimated \$100 million in total.

The bolt spacing is 5 metres, so there are about 38,000 bolts per tube, giving a total cost over 10 tubes of \$220 million.

The bolt mass is 7 kg and the maximum speed is 2.4 km/sec. Then the ambit radius is 210 metres, giving a tunnel length, including accelerator/decelerator, of about 2.5 km at each surface station. This leads to an overall cost estimate of \$470 million.

These figures sum to \$2010 million, including the bearer and tourist vehicles. Adding 20% for R&D gives a total of \$2.4 billion.

5.2 Version 2

The tube length is 352 km. The combined cost of NIB and other materials is $300 per metre. Over 10 tubes this is $1060 million. The mass of Kevlar® per tube is 3,100,000 kg, giving a total cost over 10 tubes of $2800 million. The support tubes cost an estimated $60 million in total.

The bolt spacing is 2.5 metres, so there are 140,000 bolts per tube, costing $810 million over 10 tubes.

The bolt mass is 7 kg and the maximum speed is 3 km/sec. The ambit radius is 330 metres, giving a tunnel length, including accelerator/decelerator, of about 4.5 km at each surface station. This cost is estimated at $840 million.

The sum is $6170 million. Adding 20% for R&D gives a total of $7.4 billion.

5.3 Version 3

The tube length is 1050 km. The combined cost of NIB and other materials is $300 per metre. Over 10 tubes this is $3150 million. The mass of Kevlar per tube is 10,000,000 kg, giving a total cost over 10 tubes of $9000 million. The support tubes cost an estimated $100 million in total.

The bolt spacing is 4.5 metres, so there are 233,000 bolts per tube, costing $1350 million over 10 tubes.

The bolt mass is 5 kg and the maximum speed is 11 km/sec. The ambit radius is 3.2 km, giving a tunnel length, including accelerator/decelerator, of about 35 km at each surface station. This cost is estimated at $6510 million.

The sum is $20.7 billion Adding 20% for R&D gives a total of $24.9 billion.

6. CONCLUSION

Although the costs are large, they are in line with those of other space projects. The estimated life-time cost of the Hubble space telescope, for instance, is of the order of $6 billion. Several astronomical and other scientific instruments could be installed on the space cable at much lower cost with the huge advantage of ready access for servicing and upgrades.

Stability has been one of the key concerns for the space cable, particularly lateral stability. Now that a solution has been found, a major obstacle to progress has been eliminated. There are still many open questions over such a large infrastructure project but the long-term benefits and potential cost savings for access to space are substantial.

APPENDIX: EQUATIONS GOVERNING LATERAL STABILITY

Inequalities (1) and (2) given in Section 4 are derived here.

The equation for a bolt's lateral acceleration [5] is

$$\frac{d^2 z}{dt^2} = \frac{\partial^2 z}{\partial t^2} + 2V\frac{\partial^2 z}{\partial x \partial t} + V^2 \frac{\partial^2 z}{\partial x^2} \tag{A.1}$$

The displacement along the tube is x, the lateral displacement is z, velocity is V and time is t. In a tube, let R_a be the average force per unit distance between an ascending bolt and the tube. Let the bolt's lateral displacement be z_a; its velocity is V_a and mass per metre m_a. Then the equation of motion is

$$R_a = m_a\left(\frac{\partial^2 z_a}{\partial t^2} + 2V_a\frac{\partial^2 z_a}{\partial x \partial t} + {V_a}^2 \frac{\partial^2 z_a}{\partial x^2}\right) \tag{A.2}$$

Similarly for the second tube, in which the descending bolts' velocity is $-V_d$, the mass per unit distance is m_d, the lateral displacement is z_d and the force per unit distance is R_d, the equation of motion is

$$R_d = m_d\left(\frac{\partial^2 z_d}{\partial t^2} - 2V_d\frac{\partial^2 z_d}{\partial x \partial t} + {V_d}^2 \frac{\partial^2 z_d}{\partial x^2}\right) \tag{A.3}$$

A pair of tubes is therefore subject to the sum of these forces $R_a + R_d$. In addition, there is an external force F, due mainly to wind, and a force

$$T\frac{\partial^2 z}{\partial x^2}$$

due to tension T in each tube [1]. Finally, there are internally generated forces F_{ea} and F_{ed} due to the bending compensation effected by the electromagnets in the bolts; these forces are designed to control the way the tube bends. They were introduced in [7] and are generalised here to cover the case $m_a \neq m_d$ and $V_a \neq V_d$, as happens during a launch. The equation of motion of a pair of tubes is

$$2F + F_{ea} + F_{ed} = R_a + R_d + 2\left(m_t\frac{\partial^2 z}{\partial t^2} - T\frac{\partial^2 z}{\partial x^2}\right) \tag{A.4}$$

The forces are all measured per unit distance; m_t is a tube's mass per unit distance.

Combine equations (A.2), (A.3) and (A.4) as follows.

$$\begin{aligned} 2F + F_{ea} + F_{ed} &= m_a\left(\frac{\partial^2 z_a}{\partial t^2} + 2V_a\frac{\partial^2 z_a}{\partial x \partial t} + {V_a}^2 \frac{\partial^2 z_a}{\partial x^2}\right) \\ &+ m_d\left(\frac{\partial^2 z_d}{\partial t^2} - 2V_d\frac{\partial^2 z_d}{\partial x \partial t} + {V_d}^2 \frac{\partial^2 z_d}{\partial x^2}\right) \\ &+ 2\left(m_t\frac{\partial^2 z}{\partial t^2} - T\frac{\partial^2 z}{\partial x^2}\right) \end{aligned} \tag{A.5}$$

A stable position exists in which the time derivatives are all zero and the bending forces $F_{ea} = F_{ed} = 0$. In this stable position equation (A.5) simplifies to

$$2F = m_a {V_a}^2 \frac{\partial^2 z_a}{\partial x^2} + m_d {V_d}^2 \frac{\partial^2 z_d}{\partial x^2} - 2T\frac{\partial^2 z}{\partial x^2} \tag{A.6}$$

This equation shows that the curvature can be balanced against the external force F. In this position, acceleration is eliminated. Introduce the required displacement z_r to satisfy equation (A.6).

$$\left(m_a V_a{}^2 + m_d V_d{}^2 - 2T\right)\frac{\partial^2 z_r}{\partial x^2} = 2F \qquad (A.7)$$

The bolts apply a bending compensation to a tube by means of differential forces, as described in Section 4.4. An important part of the bending compensation in a tube is B_{ca} or B_{cd}.

$$B_{ca} = \chi_a \frac{\partial^2}{\partial x \partial t}(c - c_r) + \xi_a \frac{\partial^2}{\partial x^2}(c - c_r) - \varepsilon(c - c_r) - \kappa \frac{\partial}{\partial t}(c - c_r)$$
(A.8)

The term proportional to ε causes the actual curvature

$$c = \frac{\partial^2 z}{\partial x^2}$$

to change and come into line with the required curvature

$$c_r = \frac{\partial^2 z_r}{\partial x^2} = \frac{2F}{M} \qquad (A.9)$$

There is a damping term proportional to κ. The term in

$$\chi_a \frac{\partial^2}{\partial x \partial t}(c - c_r)$$

helps to deal with the shear effect of uneven masses and velocities in a pair of tubes. The term

$$\xi_a \frac{\partial^2}{\partial x^2}(c - c_r)$$

avoids applying unnecessary bending compensation when the variations in wind and curvature are already achieving the desired effect. Similar terms

$$\chi_d \frac{\partial^2}{\partial x \partial t}(c - c_r)$$

and

$$\xi_d \frac{\partial^2}{\partial x^2}(c - c_r)$$

are added to B_{cd} in the other tube.

The aerodynamic damping even at low altitudes was found to be negligible.

Write $z' = z - z_r$. Equation (A.8) is equivalent to

$$B_{ca} = \chi_a \frac{\partial^4 z'}{\partial x^3 \partial t} + \xi_a \frac{\partial^4 z'}{\partial x^4} - \varepsilon \frac{\partial^2 z'}{\partial x^2} - \kappa \frac{\partial^3 z'}{\partial x^2 \partial t} \qquad (A.10)$$

The bolts apply a net bending compensation B_{ea} and B_{ed}, which include terms incorporating wind acceleration and rate of change of wind shear. B_{ea} is defined as follows.

$$B_{ea} = B_{ca} + \frac{m}{M}\frac{\partial^2 F}{\partial t^2} + \frac{D}{M}\frac{\partial^2 F}{\partial x \partial t} = B_{ca} + \tfrac{1}{2} m \frac{\partial^4 z_r}{\partial x^2 \partial t^2} + \tfrac{1}{2} D \frac{\partial^4 z_r}{\partial x^3 \partial t}$$
(A.11)

Here $m = m_a + m_d + 2m_t$, and $D = 2(m_a V_a - m_d V_d)$. The force F_{ea} introduced in equation (A.4) is defined as $F_{ea} = \iint B_{ea} dx dx$. F_{ed}, F_{ca} and F_{cd} are similar. Integrate equation (A.11) twice with respect to x, do the same for the other tube, and sum them to obtain a net force on a pair of tubes.

$$F_{ea} + F_{ed} = F_{ca} + F_{cd} + m\frac{\partial^2 z_r}{\partial t^2} + D\frac{\partial^2 z_r}{\partial x \partial t}$$

Hence and by integrating equation (A.10)

$$\begin{aligned} F_{ea} + F_{ed} &= m\frac{\partial^2 z_r}{\partial t^2} + D\frac{\partial^2 z_r}{\partial x \partial t} + (\chi_a + \chi_d)\frac{\partial^2 z'}{\partial x \partial t} \\ &+ (\xi_a + \xi_d)\frac{\partial^2 z'}{\partial x^2} - 2\left(\varepsilon z' + \kappa \frac{\partial z'}{\partial t}\right) \end{aligned} \qquad (A.12)$$

With the appropriate choice of origin, the constants of integration are zero. Since no displacement is wanted at high altitudes, the origin is taken at the highest point of the space cable. Whereas a tube is articulated, the bolts are not connected and each one is effectively rigid. Consequently, there is no net force on the bolts as a result of the bending correction.

Consider the lateral forces R_a and R_d per unit distance between the bolts and the tubes. The bolts have respective lateral displacements z_a and z_d, so $z - z_d$ is the distance between the tube and the bolts ascending inside it. Define R_a as a function of $z - z_a$ and of the required displacement z_r as follows.

$$\begin{aligned} R_a &= \alpha(z - z_a) + \beta\frac{\partial}{\partial t}(z - z_a) \\ &+ m_a\left(\frac{\partial^2 z_r}{\partial t^2} + 2V_a\frac{\partial^2 z_r}{\partial x \partial t} + V_a{}^2\frac{\partial^2 z_r}{\partial x^2}\right) \end{aligned} \qquad (A.13)$$

The purpose of this definition of R_a is to keep the bolts near but not crashing into the sides of the tubes. The constant α governs this attraction or repulsion; β is for damping the consequent motion. The terms in m_a supply the centripetal force needed to pull the bolts round a curve and respond to acceleration of the tube.

Write $z'_a = z_a - z_r$ and $z'_d = z_d - z_r$. Then

$$\begin{aligned} R_a &= \alpha(z' - z'_a) + \beta\frac{\partial}{\partial t}(z' - z'_a) \\ &+ m_a\left(\frac{\partial^2 z_r}{\partial t^2} + 2V_a\frac{\partial^2 z_r}{\partial x \partial t} + V_a{}^2\frac{\partial^2 z_r}{\partial x^2}\right) \end{aligned} \qquad (A.14)$$

$$\begin{aligned} R_d &= \alpha(z' - z'_d) + \beta\frac{\partial}{\partial t}(z' - z'_d) \\ &+ m_d\left(\frac{\partial^2 z_r}{\partial t^2} - 2V_d\frac{\partial^2 z_r}{\partial x \partial t} + V_d{}^2\frac{\partial^2 z_r}{\partial x^2}\right) \end{aligned} \qquad (A.15)$$

The equations of motion of the bolts are equations (A.2) and (A.3). They can be written

$$\begin{aligned} R_a &= m_1\left(\frac{\partial^2 z'_a}{\partial t^2} + 2V_a\frac{\partial^2 z'_a}{\partial x \partial t} + V_a{}^2\frac{\partial^2 z'_a}{\partial x^2}\right) \\ &+ m_a\left(\frac{\partial^2 z_r}{\partial t^2} + 2V_a\frac{\partial^2 z_r}{\partial x \partial t} + V_a{}^2\frac{\partial^2 z_r}{\partial x^2}\right) \end{aligned} \qquad (A.16)$$

$$R_d = m_d\left(\frac{\partial^2 z_d'}{\partial t^2} - 2V_d\frac{\partial^2 z_d'}{\partial x\partial t} + V_d{}^2\frac{\partial^2 z_d'}{\partial x^2}\right) + m_d\left(\frac{\partial^2 z_r}{\partial t^2} - 2V_d\frac{\partial^2 z_r}{\partial x\partial t} + V_d{}^2\frac{\partial^2 z_r}{\partial x^2}\right) \quad (A.17)$$

Hence and from equation (A.14) and (A.15),

$$\alpha(z' - z_a') + \beta\frac{\partial}{\partial t}(z' - z_a') = m_a\left(\frac{\partial^2 z_a'}{\partial t^2} + 2V_a\frac{\partial^2 z_a'}{\partial x\partial t} + V_a{}^2\frac{\partial^2 z_a'}{\partial x^2}\right) \quad (A.18)$$

$$\alpha(z' - z_d') + \beta\frac{\partial}{\partial t}(z' - z_d') = m_d\left(\frac{\partial^2 z_d'}{\partial t^2} - 2V_d\frac{\partial^2 z_d'}{\partial x\partial t} + V_d{}^2\frac{\partial^2 z_d'}{\partial x^2}\right) \quad (A.19)$$

Substitute equations (A.7), (A.12), (A.16) and (A.17) in (A.4).

$$\begin{aligned} &m\frac{\partial^2 z_r}{\partial t^2} + D\frac{\partial^2 z_r}{\partial x\partial t} + M\frac{\partial^2 z_r}{\partial x^2} + (\chi_a + \chi_d)\frac{\partial^2 z'}{\partial x\partial t} \\ &+ (\xi_a + \xi_d)\frac{\partial^2 z'}{\partial x^2} - 2\left(\varepsilon z' + \kappa\frac{\partial z'}{\partial t}\right) = \frac{\partial^2}{\partial t^2}(m_a z_a' + m_d z_d') \\ &+ 2\frac{\partial^2}{\partial x\partial t}(m_a V_a z_a' - m_d V_d z_d') + \frac{\partial^2}{\partial x^2}(m_a V_a^2 z_a' + m_d V_d^2 z_d') \\ &+ (m_a + m_d)\frac{\partial^2 z_r}{\partial t^2} + D\frac{\partial^2 z_r}{\partial x\partial t} + (m_a V_a^2 + m_d V_d^2)\frac{\partial^2 z_r}{\partial x^2} \\ &+ 2\left(m_t\frac{\partial^2 z}{\partial t^2} - T\frac{\partial^2 z}{\partial x^2}\right) \end{aligned} \quad (A.20)$$

This equation simplifies as follows.

$$\begin{aligned} &\frac{\partial^2}{\partial t^2}(m_a z_a' + m_d z_d') + 2\frac{\partial^2}{\partial x\partial t}(m_a V_a z_a' - m_d V_d z_d') \\ &+ \frac{\partial^2}{\partial x^2}(m_a V_a^2 z_a' + m_d V_d^2 z_d') - (\chi_a + \chi_d)\frac{\partial^2 z'}{\partial x\partial t} \\ &- (\xi_a + \xi_d)\frac{\partial^2 z'}{\partial x^2} + 2\left(\varepsilon z' + \kappa\frac{\partial z'}{\partial t} + m_t\frac{\partial^2 z'}{\partial t^2} - T\frac{\partial^2 z'}{\partial x^2}\right) = 0 \end{aligned} \quad (A.21)$$

It is not necessary to solve these equations; it is sufficient to show that they converge to a stable value over time. For this, the Laplace transform is used, [10] which is a generalisation of the Fourier transform allowing complex exponents. Let

$$Z'(\psi, \varphi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z'(t, x)e^{-\psi t - \varphi x}dxdt \quad (A.22)$$

so that

$$z'(t, x) = \int\int_{c-i\infty}^{c+i\infty} Z'(\psi, \varphi)e^{\psi t + \varphi x}d\varphi d\psi \quad (A.23)$$

Both ψ and φ are complex in general. The limits of the integration with respect to ψ in equation (A.23) have not been specified but it should follow a path in the complex plane of ψ that takes in $\pm i\infty$. The constant c can be selected for convenience, by Cauchy's theorem of complex integration. Similarly, the Laplace transforms of z_a' and z_d' are

$$Z_a'(\psi, \varphi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z_a'(t, x)e^{-\psi t - \varphi x}dxdt \quad (A.24)$$

$$Z_d'(\psi, \varphi) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z_d'(t, x)e^{-\psi t - \varphi x}dxdt \quad (A.25)$$

Analogously to the Fourier transform, the Laplace transforms of

$$\frac{\partial z'}{\partial t}$$

and

$$\frac{\partial z'}{\partial x}$$

are $\psi Z'$ and $\varphi Z'$ respectively.

The transforms of equations (A.18) and (A.19) are

$$r(Z' - Z_a') = x_a Z_a'$$
$$r(Z' - Z_d') = x_d Z_d'$$

Here $r = a + \beta\psi$,

$$x_a = m_a\left(\psi^2 + 2V_a\psi\varphi + V_a{}^2\varphi^2\right)$$

and

$$x_d = m_d\left(\psi^2 - 2V_d\psi\varphi + V_d{}^2\varphi^2\right).$$

Therefore

$$(1 + \frac{x_a}{r})Z_a' = Z' \quad (A.26)$$

$$(1 + \frac{x_d}{r})Z_d' = Z' \quad (A.27)$$

A large value of $r = \alpha + \beta\psi$ makes $Z_a' \approx Z' \approx Z_d'$, which is essential to prevent the bolts crashing into the walls of the tubes.

The transform of equation (A.21) is

$$\begin{aligned} &x_a Z_a' + x_d Z_d' - (\chi_a + \chi_d)\varphi\psi Z' - (\xi_a + \xi_d)\varphi^2 Z' \\ &+ 2(m_t\psi^2 - T\varphi^2 + \varepsilon + \kappa\psi)Z' = 0 \end{aligned} \quad (A.28)$$

On the assumption of a large r, equation (A.28) is, to a first approximation,

$$\begin{aligned} &x_a + x_d - (\chi_a + \chi_d)\varphi\psi - (\xi_a + \xi_d)\varphi^2 \\ &+ 2(\varepsilon + \kappa\psi + m_t\psi^2 - T\varphi^2) = 0 \end{aligned} \quad (A.29)$$

Write equation (A.29)

$$m\psi^2 + h\varphi\psi + N\varphi^2 + 2\kappa\psi + 2\varepsilon = 0 \quad (A.30)$$

Here $m = m_a + m_d + 2m_t$ is the combined mass per unit length of a pair of bolts and tubes,

$$h = 2(m_a V_a - m_d V_d) - (\chi_a + \chi_d) \quad \text{(A.31)}$$

$$N = m_a V_a^2 + m_d V_d^2 - 2T - (\xi_a + \xi_d) \quad \text{(A.32)}$$

The value of h will be kept small by setting

$$\chi_a = 2(m_a V_a - m'V') + h_a$$

and

$$\chi_d = 2(m'V' - m_d V_d) + h_d,$$

where $m'V'$ are constant typical values of $m_a V_a$ and $m_d V_d$, while h_a and h_d are small estimating errors in the bolts ($h = h_a + h_d$). It is not safe to assume a small value for N. Setting

$$\xi_a = m_a V_a^2 - (N_a + T)$$

and

$$\xi_d = m_d V_d^2 - (N_d + T)$$

with $N = N_a + N_d$ involves much larger quantities than χ_a and χ_d. These values are used to set the correct values for B_{ca} in equation (A.8) and for B_{cd}, where they are multiplied by the spatial derivative of wind shear, which has to be estimated. Thus the value that the bolts actually apply is bound to involve some degree of error (see Section 4.2).

Equation (A.30) is quadratic in ψ with solution

$$2m\psi = -(2\kappa + h\varphi) \pm \sqrt{(2\kappa + h\varphi)^2 - 4m(2\varepsilon + N\varphi^2)} \quad \text{(A.33)}$$

Since ψ is a function of ϕ, equation (A.23) simplifies to

$$z'(t,x) = \int_{c-i\infty}^{c+i\infty} Z'(\varphi) e^{\psi t + \varphi x} d\varphi$$

The constant $c = 0$ is chosen. Writing $\varphi = i\omega$ where $i^2 = -1$, equation (A.33) becomes

$$2m\psi = -(2\kappa + ih\omega) \pm \sqrt{(2\kappa + ih\omega)^2 - 4m(2\varepsilon - N\omega^2)} \quad \text{(A.34)}$$

For stability, the real part of ψ needs to be negative over the range of possible values of ω. Let

$$\sqrt{(2\kappa + D\phi)^2 - 4m(2\varepsilon + N\phi^2)} = a + ib = \sqrt{p + iq}$$

Then the real part of ψ is negative if $\kappa > 0$ and $a < 2H\kappa$ for a constant $H < 1$. H and κ govern the speed of convergence. Now $a^2 - b^2 = p$ and $2ab = q$, so $4a^4 - q^2 = 4a^2 p$. Thus $a < H\kappa$ if

$$16H^4\kappa^4 > 4H^2\kappa^2 p + \tfrac{1}{4}q^2 \quad \text{(A.35)}$$

Discarding terms in h^2,

$$p = 4(\kappa^2 - 2m\varepsilon) - (h^2 - 4mN)\omega^2 \approx 4(\kappa^2 - 2m\varepsilon + mN\omega^2)$$

$$q^2 = (4\kappa h\omega)^2 \approx 0$$

So inequality (A.35) is true if

$$H^2\kappa^2 > \kappa^2 - 2m\varepsilon + mN\omega^2 \quad \text{(A.36)}$$

That is

$$2\varepsilon > N\omega^2 + (1 - H^2)\frac{\kappa^2}{m} \quad \text{(A.37)}$$

Write $i\omega = 2\pi i/W$ to see that values of ω represent standing waves in the cable of wavelength W. The maximum value of ω is determined by the shortest wavelength W_S. Then inequality (A.37) may be written

$$\varepsilon > N\frac{2\pi^2}{W_S^2} + (1 - H^2)\frac{\kappa^2}{2m} \quad \text{(A.38)}$$

This is inequality (1) in Section 4. Equation (A.33) also requires

$$\kappa > 0 \quad \text{(A.39)}$$

This is inequality (2).

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